Implementation of subgrid approaches accounting for unresolved topography in MPAS-Ocean

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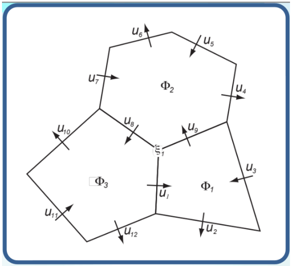
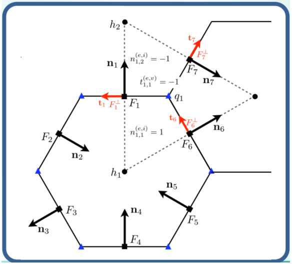
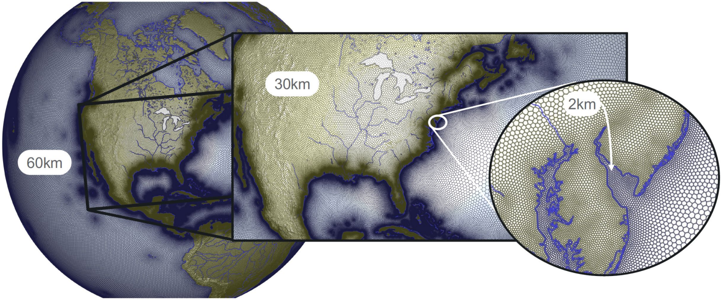
- Unstructured-grid ocean model
- Incompressible NS with the Boussinesq and hydrostatic approximation
- *Vertical*: ALE, z, z-star, σ coordinates
- *Method*: Finite Volume schemes
  - Maintains geostrophic balance.
  - Conserves energy
  - Maintain Lagrangian conservation of potential vorticity
- *Horizontal*: Voronoi tessellation, C-grid
- *Time stepping*: 2D barotropic/3D baroclinic mode splitting, RK4





[1] Ringler, Thuburn, Klemp, Skamarock, JCP, 2010; Petersen et al, Ocean Modelling, 2015

MPAS-Ocean

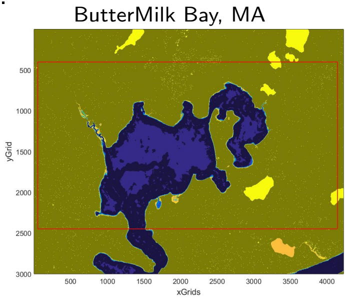


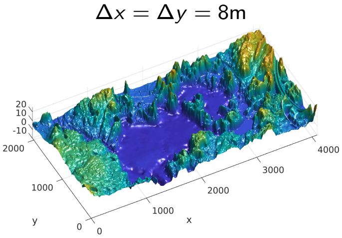
- Currently, MAPS-O global ocean component of E3SM resolves the ocean and its coast no finer than 6 km[2].
- Resolution in the coastal zones will not be finer 2 km in the foreseeable future.
- Potentially insufficient to capture the hydrodynamics of the terrestrial-aquatic interface connecting the upland hydrology and the ocean through the hydraulic conveyances penetrating the coastal floodplain.

[2] Petersen et al., J. Adv. Model. Earth Syst., 2019

Subgrid modelling with unresolved topography

- The bottom profile is typically approximated on the *grid level*.
- A large amount of topographical detail could be lost in the low resolution case.





3 see Casulli, IJNMF, 2009; Stelling, Water Manag, 2012; Kennedy et al., Ocean Modelling, 2019

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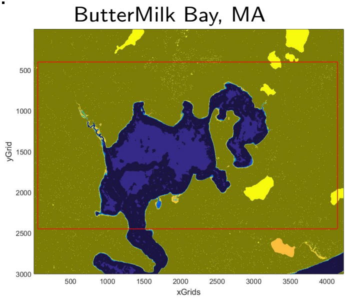
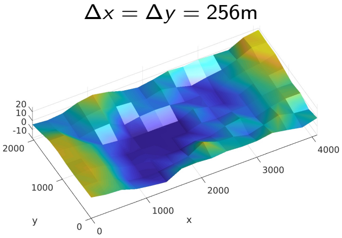




- Subgrid approaches[3], which permit the use the subgrid bathymetric details, have been shown to be very effective in improving a coarse-grid model accuracy.

[3] see Casulli, IJNMF, 2009; Stelling, Water Manag, 2012; Kennedy et al., Ocean Modelling, 2019

Implementation of subgrid correction

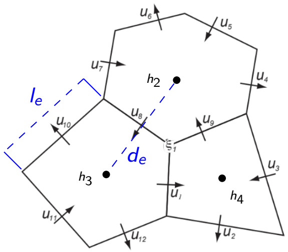
- A single-layer mode of MPAS-O:

$$\frac{\partial h}{\partial t} + \nabla \cdot (h\boldsymbol{u}) = 0.$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + q\boldsymbol{u}^{\perp} = -\nabla g\left(\eta + K\right) - \frac{C_f}{h}|\boldsymbol{u}|\boldsymbol{u} + \frac{1}{h}\boldsymbol{F},$$

where $q = \mathbf{k}\cdot\nabla\times\boldsymbol{u} + f$, $\boldsymbol{u}^{\perp} = \mathbf{k}\times\boldsymbol{u}$, $K = |\boldsymbol{u}^2|/2$, $\eta = h - b$

- MPAS-O semi-discrete discretization on polygonal orthogonal C-grid.



$$\frac{d\ h_i}{dt} = -\frac{1}{A_i}\sum_{e\in EC(i)} u_{n,e}\ h_e\ l_e$$

$$\frac{du_{n,e}}{dt} = \frac{g}{d_e}\sum_{i\in CE(e)} -n_{e,i}\eta_i - \frac{C_f}{\overline{h}_e}|\boldsymbol{u}_e|u_{n,e} + \frac{1}{\overline{h}_e}F_{n,e} + \text{TRiSK\_adv\_scheme} + \cdots$$

Implementation of subgrid correction[4]

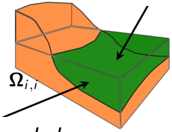
- In MPAS-O, as usual,

$$h_i = \eta_i + b_i, \;\; b_i = \frac{1}{A_i}\int_{A_i} b(\boldsymbol{x})d\boldsymbol{x}$$

- To correct mass,

$$h_i(\eta_i) = \frac{1}{A_i}\int_{A_i} \max(\eta_i + b(\boldsymbol{x}), 0)d\boldsymbol{x}$$

$$h_e(\eta_e) = \frac{1}{l_e}\int_{l_e} \max(\eta_e + b(\boldsymbol{x}), 0)d\boldsymbol{x}$$



- Assume constant friction slope flow,

$$C_f = \frac{1}{A_i}\frac{\left[\int_{A_i} \max(0, \eta_i + b(\boldsymbol{x}))d\boldsymbol{x}\right]^3}{\left[\int_{A_i} \max(0, \eta_i + b(\boldsymbol{x})^{\frac{3}{2}}\sqrt{1/c_f}d\boldsymbol{x}\right]^2}.$$

This 'bulk' C_f is well-suited for channel-like flow.

[4] Kennedy et al, Ocean modelling, 2019; Casulli, IJNFM, 2009

Parabolic bowl problem [Thacker, 1981]

- Frictionless rotating basin:

$$b(x,y) = h_0 \left(1 - \frac{r^2}{L^2}\right)$$

where $r = \sqrt{x^2 + y^2}$, $L = Const$.

- Initial condition:

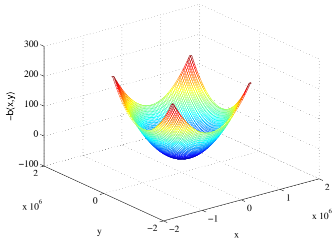
$$\boldsymbol{u}(\boldsymbol{x},0) = \frac{1}{2C_0}\left[f(\sqrt{1-C^2} - C_0)\right](-y,x)$$

$$H(\boldsymbol{x},0) = h_0\left[\frac{\sqrt{1-C^2}}{C_0} - \frac{r^2}{L^2}\left(\frac{1+C}{(C_0}\right)\right]$$

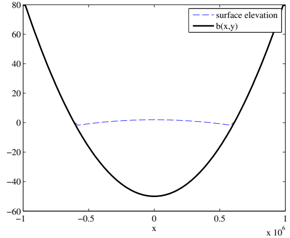
$$\eta(\boldsymbol{x},0) = H(\boldsymbol{x},0) - b(\boldsymbol{x})$$

- $C = \dfrac{(h_0+\zeta_0)^2 - h_0^2}{(h_0+\zeta_0)^2 + h_0^2}, \zeta_0 = \eta(\mathbf{0},0),$
  $C_0 = 1 - C$
- $L = \sqrt{\dfrac{8gh_0}{\omega^2 - f^2}}$ for a given ω

Bed profile

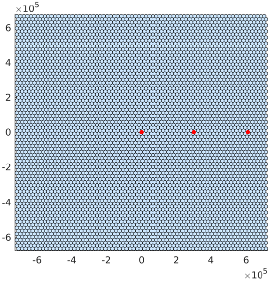


Initial surface elevation



Parabolic bowl problem

$h \sim 20$km



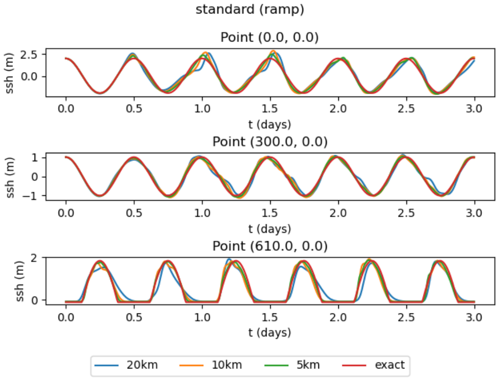
Parameters:

$h_0 = 50$m, $\zeta_0 = 2$m,
$f = 1.03 \times 10^{-3}$, $\omega = 2\pi/12$ (hr^{-1})

Numerical details:

- $h = 20$ km ($\Delta t = 200$s), 10 km ($\Delta t = 100$s), and 5km ($\Delta t = 50$).
- Subgrid bathy $h_s \sim 0.5/\sqrt{3}$ km.

Time series: Standard



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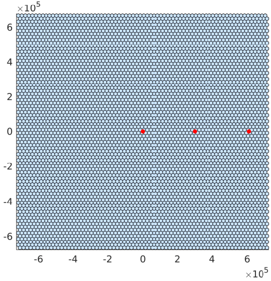

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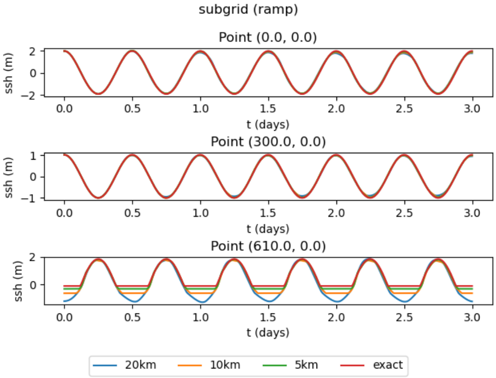
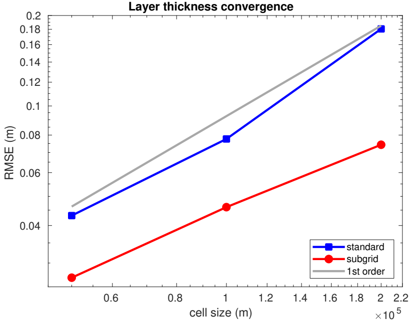
TIME SERIES: SUBGRID

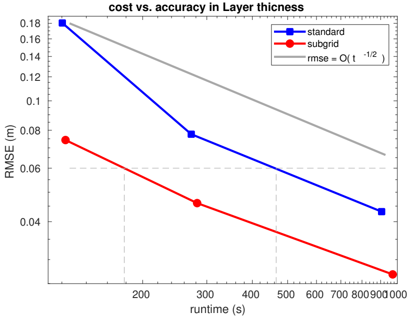


Parabolic bowl problem

cell size *vs.* error



runtime *vs.* error



- Error $= \sqrt{\frac{1}{N} \sum_i (H_{h,i} - H_{ex,i})^2}$

Parabolic bowl problem

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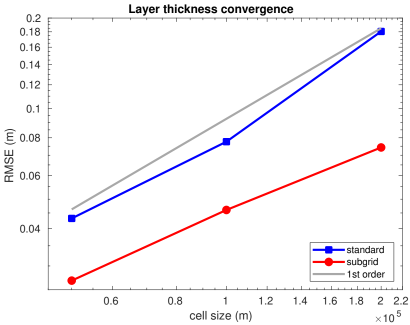

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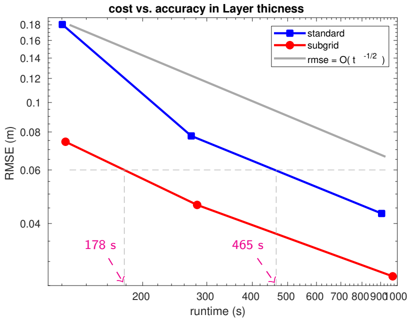

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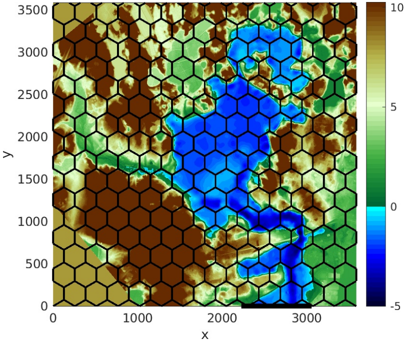
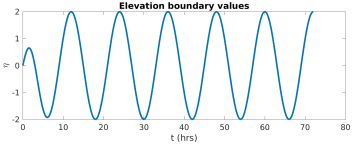
Buttermilk

LiDAR topography: Buttermilk bay, MA



Surface elevation imposed on the inlet



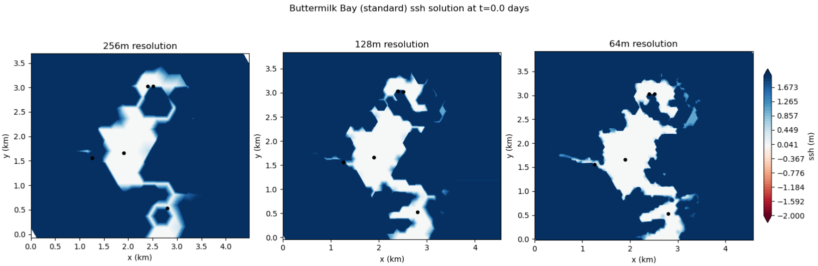
Problem:

- Flow is driven by elevation boundary
- Chezy coefficient $= 0.017$

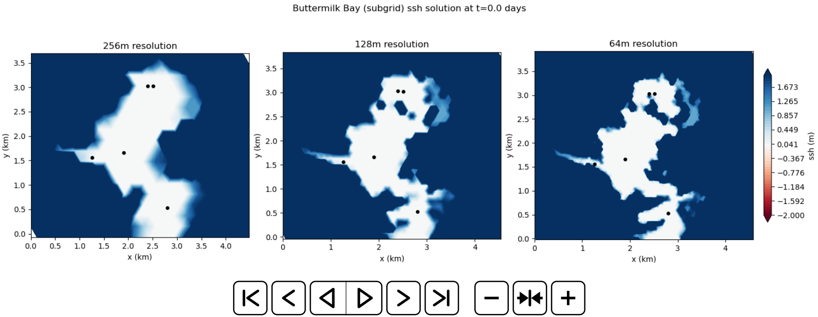
Numerical method:

- RK4. $\Delta t = 10$s.
- Single layer.
- Uniform meshes: $h \sim 256$m-64m.
- Subgrid bathy $h_s \sim 6.4/\sqrt{3}$ m

Buttermilk bay: standard

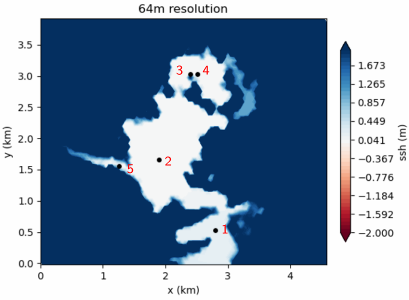


Buttermilk bay: subgrid

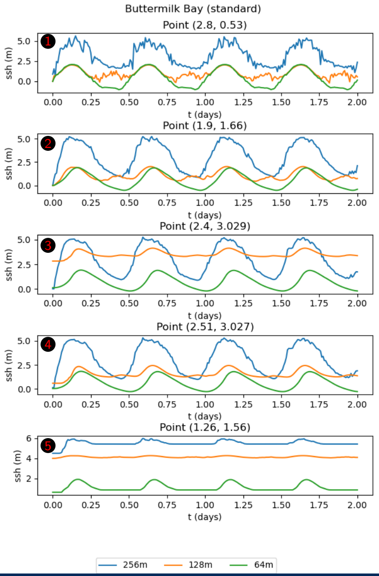


Buttermilk bay: time series

STATION LOCATIONS



$\eta(t)$



Buttermilk bay: time series

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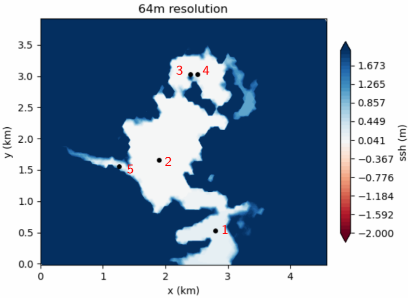

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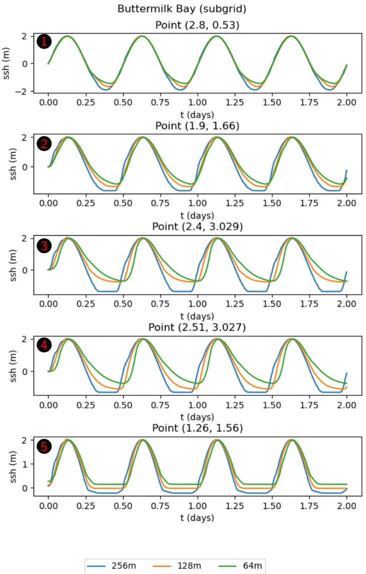
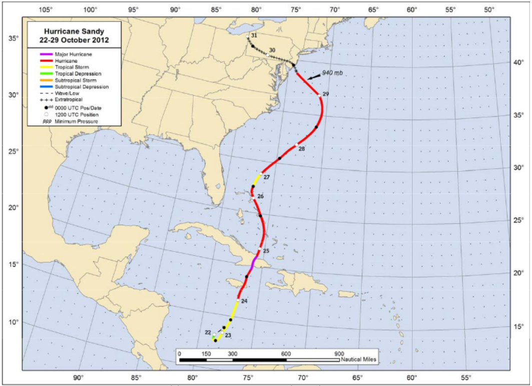


Storm Tide: Hurricane Sandy (2012)



source: `https://www.weather.gov/okx/hurricanesandy5year`

- SO12to60 mesh: 2.7×10^6 cells, 8.24×10^6 edges.
- Subgrid inset from SC to ME with CRM DEM ($\approx$ 30m res).
- Tides, wind stress and atm pressure (CFSv2), log law for bottom drag.
- $\Delta t = 5$ seconds. Run period 10/10/2012–11/02/2012.

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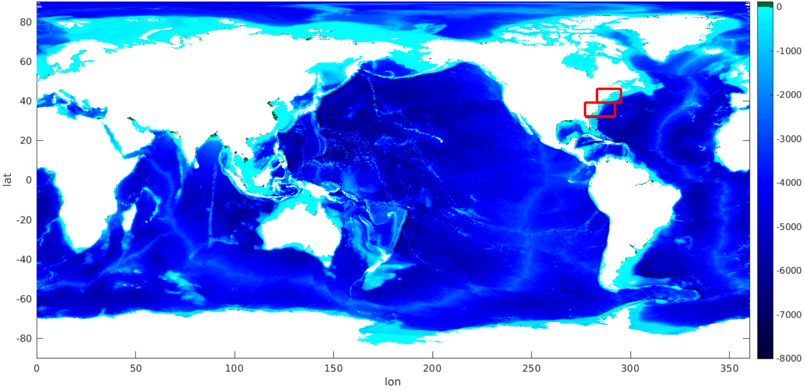

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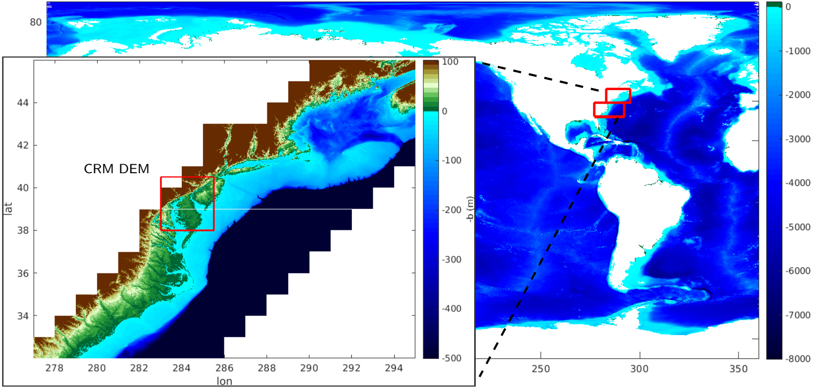

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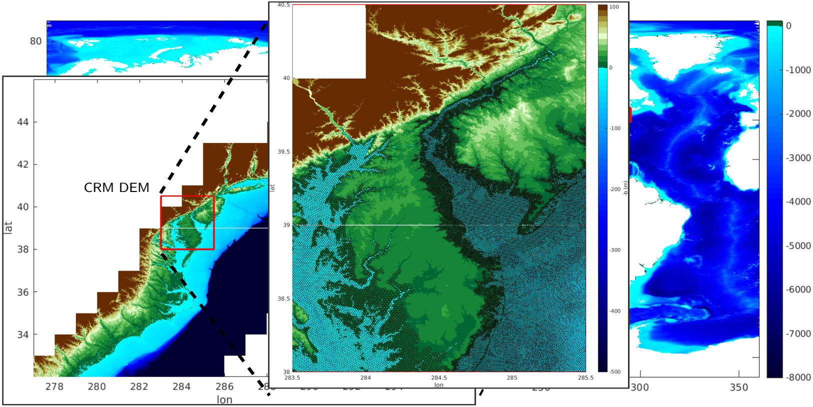

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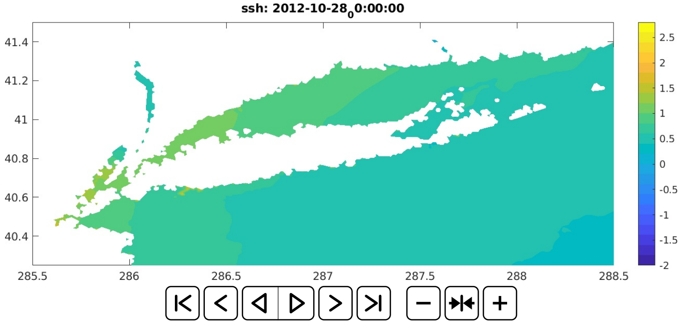
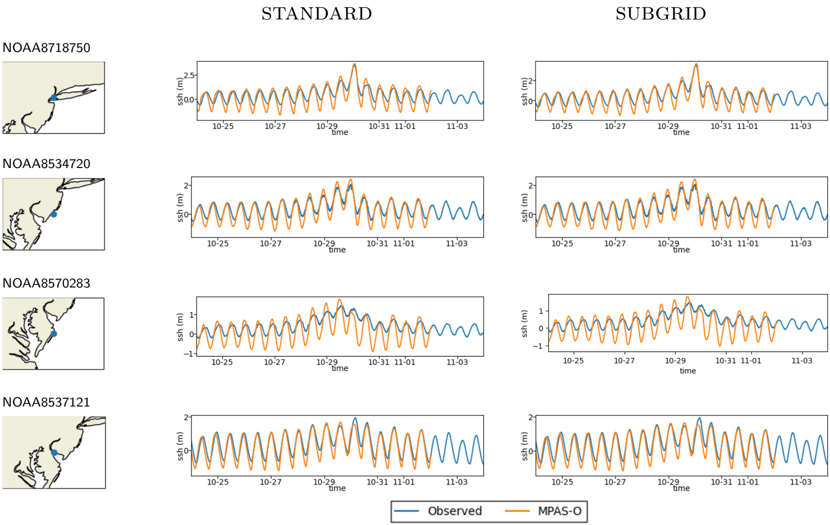
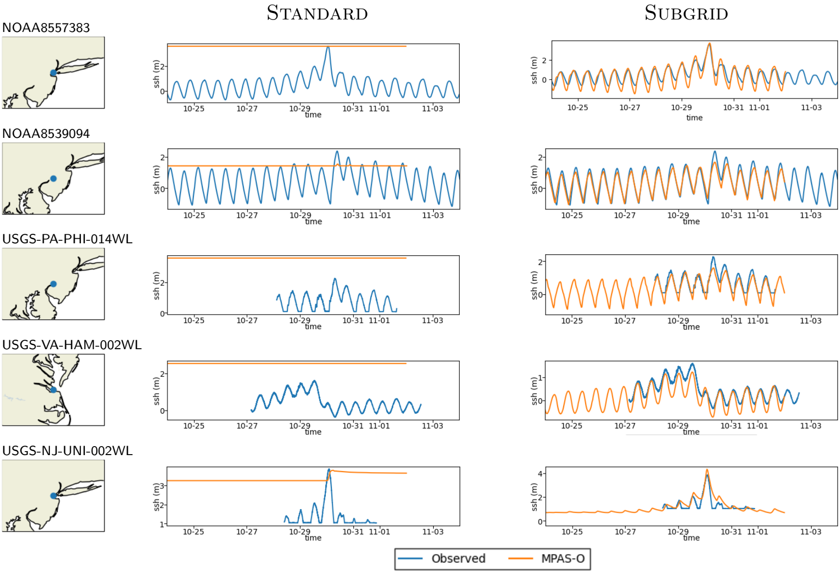
Hurricane Sandy: SSH over NY coast, subgrid run



Hurricane Sandy: SSH validation, “open water” gauges

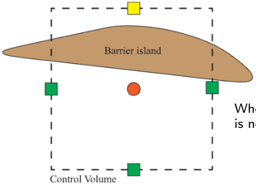


Hurricane Sandy. SSH validation, “coastal” gauges



Summary

- The subgrid corrections using high resolution bathymetric and bed roughness data are implemented in the single-layer mode of the MPAS-Ocean.
- Verification and validation using idealized non-trivial test case with analytical solution clearly and real world applications show increased accurracy and gain cost-per-accurcy of the subgrid MPAS-Ocean solution.
- Simulations of global storm-tide with compreshsive forcing terms, including tides, self attraction and loading, internal tide parameterization, winds, and ice cap are underway!
- Although can be mitigated to some level by an unstructured grid, connectivity issues remain one of the most challenging aspects of the subgrid methods.



When not fully inundated, one indepdent ssh is not enough to capture the processes.